Fair division under asymmetric information

van Damme, E.E.C.

Published in:
Rational interaction

Publication date:
1992

Document Version
Publisher's PDF, also known as Version of record

Fair division under asymmetric information

van Damme, Eric

Publication date:
1992

Citation for published version (APA):
van Damme, E. E. C. (1992). Fair division under asymmetric information. (Reprint series / CentER for Economic Research; Vol. 85). Unknown Publisher.

CentER for Economic Research

REPRINT

Fair Division under Asymmetric Information

by

Eric van Damme

Reprinted from R. Selten (ed.),
Rational Interaction - Essays in Honor of
John C. Harsanyi, Berlin/Heidelberg:
Springer-Verlag, 1992

Reprint Series
no. 85

TILBURG UNIVERSITY

Address: Warandelaan 2, P.O. Box 90153, 5000 LE Tilburg, The Netherlands
Phone : +31 13 663050
Telex : 52426 kub nl
Telefax: +31 13 663066
E-mail : "center@htikub5.bitnet"

ISSN 0924-7874

1992

CentER for Economic Research REPRINT

Fair Division under Asymmetric Information

by

Eric van Damme

Reprinted from R. Selten (ed.),
Rational Interaction - Essays in Honor of
John C. Harsanyi, Berlin/Heidelberg:
Springer-Verlag, 1992

Reprint Series
no. 85

Fair Division under Asymmetric Information

Eric van Damme

CentER, Tilburg University

P.O. Box 90153, 5000 LE Tilburg, The Netherlands

Abstract: This paper considers the situation in which a single indivisible object has to be allocated to one person out of a group whose members all have equal rights to it. Different persons value the object differently and each person only knows his own value exactly. The question is who should get the object and by how much this person should compensate the others in order to guarantee a fair and efficient allocation. After having shown that several well-known methods perform unsatisfactory, we derive an impossibility theorem showing that some classical fair division methods cannot be implemented when there is incomplete information. Finally, we give examples of mechanisms that do guarantee fair and efficient outcomes.

1 Introduction

The problem of how to divide an indivisible object among a group of persons who all have equal rights to it arises, for example, in divorce settlements and in inheritance situations in which there are equivalent heirs but there is no will. In a business setting, the problem arises in the dissolving of joint ventures. In this paper[1] we will assume that side payments between the parties are possible and we will try to answer the question of who should get the object and by how much each of the other players should be compensated in order to obtain (ex post) a fair and efficient allocation. Efficiency implies that the object should be allocated to the person who values is most. The fairness criterion we will employ is the one proposed in Foley (1967): It is required that the final allocation be envy free, i.e. no player should envy another, each player should prefer what he himself receives above what any other player receives.

There exists an extensive literature on the fair division problem[2]. In this literature various concepts of fairness have been proposed and several fair division methods, such as divide and

[1] Most of the material in this paper is taken from the unpublished working paper Van Damme (1985).

[2] For example Crawford (1977), Crawford and Heller (1979), Kuhn (1967), Luce and Raiffa (1957, Chapter 14), Pazner and Schmeidler (1978), Samuelson (1980), Steinhaus (1948) and Varian (1974).

choose, random allocation followed by bargaining, and auctioning the object followed by an equal division of the revenue, have been analyzed. In most of the literature, attention has been restricted to the case of complete information[3]. Frequently, however, it will be the case that each player, although he may know exactly how much he himself values the object, has only somewhat vague (probabilistic) information about his opponents' values. This opens up the possibility for strategic manipulation: A person might pretend that he values the object more (or less) than he actually does in order to obtain a more favorable outcome. Our aim in this paper is to study the consequences of such strategic behavior in fair division situations. Specifically we will investigate whether it is possible to obtain fair and efficient outcomes when players use their private information strategically.

Recently, considerable attention has been devoted to the study of bargaining under asymmetric information[4]. In this context the consequences of strategic behavior have been thoroughly investigated, and it has been shown that strategic behavior may prevent an ex post efficient outcome being reached. Although the insights generated by the bargaining literature are highly relevant for the problem at hand (indeed we will make extensive use of them), there are at least two novel aspects in the fair division problem. First of all we will see that division methods that treat the players asymmetrically aggravate the incentive problems. Methods that preserve the symmetry of the players are superior, they yield higher payoffs. As a consequence it is not desirable to reduce the division problem to a bargaining problem by first allocating the property rights. Secondly, in the bargaining literature, attention has been restricted to the efficiency aspect, questions of fairness have not been considered.

It should be noted that in the more abstract (cooperative) papers on games with incomplete information by Harsanyi and Selten (1972) and Myerson (1979, 1984) some of the axioms are based on equity considerations. However, these papers make the fundamental assumption that all that matters are the expected utilities at the interim stage, i.e. at the point in time where each player knows his own value but does not yet know those of his opponents. In contrast, in the present study our interest is also in the point in time where all values have been revealed as we want to guarantee that ex post there is no envy. Hence, the crucial parameters for our study are

[3]Exceptions are Güth (1986), Güth and Van Damme (1986) and Lyon (1986). In some older papers it is merely pointed out that the proposed methods are vulnerable to strategic manipulation, there is no analysis of its consequences. Dubins (1977) has shown that minmax strategies imply truthful revelation of values in the Steinhaus procedure.

[4]See Kennan and Wilson (1990) for a recent survey.

the actual, ex post, utilities.

The remainder of the paper is organized as follows. After having introduced some basic concepts in Section 2, we study, for the special case in which there are only two participants, some well-known division methods in Section 3. It is shown that methods that keep players in symmetric positions (such as auctions) outperform asymmetric division methods (such as divide and choose), but that also auctions may yield outcomes that are not envy free. In Section 4 we show that all ex post efficient mechanisms are equivalent at the interim stage (i.e. they generate the same expected utilities) and that random allocation is the worst possible mechanism. Section 5 shows that several 'classical' fair division methods cannot be implemented when there is incomplete information, and Section 6 gives an example of a mechanism that always generates fair and efficient outcomes. Section 7 investigates whether positive results can also be obtained if one requires that equilibria be in dominant strategies and Section 8 offers a brief conclusion.

2 The Division Problem

We consider a situation in which a single indivisible object has to be allocated to one person from a group of n. All players have equal rights to the object, side payments are possible, and each player is risk neutral and has a utility function that is additively separable in money and the object. Hence, if player i's value of the object is v_i and if t_i is the monetary transfer that this player pays, then his utility is $u_i = v_i - t_i$ if he gets the object while his utility is $u_i = -t_i$ if he doesn't get it. Each player's valuation is known privately, but it is common knowledge that all values are drawn[5] independently from a distribution F with support $[\underline{v}, \bar{v}]$. We assume that the density f is positive and continuous and without loss of generality we take $\underline{v} = 0$ and $\bar{v} = 1$. Hence, each player indeed values the object.

The problem is to determine which player should get the object and by how much he should compensate his partners so as to guarantee that the final allocation is both fair and efficient. A mechanism is a game form specifying the allocation rules. Formally, a *mechanism* is a tuple $\mu = < A, p, t >$, where

$$A = A_1 \times \ldots \times A_n \text{ with } A_i \text{ being the (nonempty) set of pure strategies of player } i, \qquad (2.1)$$

[5]Hence, we follow the seminal idea from Harsanyi (1967-8) to convert a situation with incomplete information to one with asymmetric information. The assumption of independence simplifies the problem, we haven't analyzed the case where values are correlated.

$p = (p_1, \ldots, p_n)$ with $p_i : A \to [0,1]$ and $\sum_i p_i(a) = 1$ for all $a \in A$. ($p_i(a)$ is the probability that i receives the object if a is chosen), and (2.2)

$t = (t_{ij})_{i,j}$ with $t_{ij} : A \to \mathbf{R}$ and $\sum_j t_{ij}(a) = 0$ for all i and all $a \in A$. ($t_{ij}(a)$ is the monetary transfer that j has to pay (to a mediator) in case $a \in A$ is chosen and the object is allocated to i). (2.3)

Note that in (2.2) we require that the object be allocated under all circumstances and that in (2.3) it is required that the players' books always balance. In Section 7 the latter constraint is relaxed by allowing the mediator to act as a clearing house whose books have to balance only on average rather than for each value combination.

Given the mechanism μ, a (pure) strategy for player i is a map $\sigma_i : [0,1] \to A_i$, and a strategy combination σ is an n-tuple of strategies, one for each player. We now introduce some additional notation. For a value vector v we write v^h for the highest value and v^s for the second highest value. We write $a = (a_{-i}, a_i)$, $v = (v_{-i}, v_i)$, $\sigma = (\sigma_{-i}, \sigma_i)$ and $a \backslash a_i' = (a_{-i}, a_i')$. Furthermore, $\sigma(v) = (\sigma_1(v_1), \ldots, \sigma_n(v_n))$ and $dF(v) = dF(v_1) \ldots dF(v_n)$. Player i's expected transfer when a is chosen is denoted by $t_i(a)$

$$t_i(a) = \sum_k p_k(a) t_{ki}(a) \tag{2.4}$$

If player i's opponents play according to σ while player i himself chooses action a_i, then his expected transfer is

$$T_i^\sigma(a_i) = \int t_i(\sigma(v) \backslash a_i) \, dF(v) \tag{2.5}$$

while the probability that he receives the object is

$$P_i^\sigma(a_i) = \int p_i(\sigma(v) \backslash a_i) \, dF(v) \tag{2.6}$$

In this case, if players i's value is v_i, then his expected utility is

$$U_i^\sigma(a_i;v_i) \;=\; v_iP_i^\sigma(a_i)-T_i^\sigma(a_i) \tag{2.7}$$

The strategy combination σ is a (Bayesian Nash) *equilibrium* of the mechanism μ if for all i and all v_i

$$\sigma_i(v_i) \in \arg\max_{a_i} U_i^\sigma(a_i;v_i) \tag{2.8}$$

A mechanism is said to be a direct mechanism if $A_i = [0,1]$ for all i, i.e. players are asked to report their values. A direct mechanism is said to be *incentive compatible* if truthtelling (i.e. $\bar{\sigma}_i(v_i) \equiv v_i$) is an equilibrium. Note that if σ is an equilibrium of the mechanism μ, then the direct mechanism $\tilde{\mu}$ determined by $\tilde{p}_i(v) = p_i(\sigma(v))$ and $\tilde{t}_{ij}(v) = t_{ij}(\sigma(v))$ is incentive compatible and leads to the same allocation. Hence, the restriction to incentive compatible direct mechanisms is without loss of generality. (This is the so-called *revelation principle*, see e.g. Myerson (1979).) For an incentive compatible direct mechanism μ, we simplify notation by writing $P_i(v_i), T_i(v_i)$ and $U_i(v_i)$ instead of $P_i^{\bar{\sigma}}(v_i), T_i^{\bar{\sigma}}(v_i)$ and $U_i^{\bar{\sigma}}(v_i;v_i)$, resp. where $\bar{\sigma}$ denotes the strategy of truthtelling ($\bar{\sigma}_i(v_i) \equiv v_i$ for all i).

We conclude this section by specifying three additional conditions that we want mechanisms to satisfy. The requirements will be formulated only for direct mechanisms. An indirect mechanism μ is said to satisfy the requirements if it has an equilibrium σ which is such that the direct mechanism $\tilde{\mu} = \mu \circ \sigma$ satisfies them. First of all, since ex ante the players are in symmetrical positions we want the mechanism to be *symmetric*, i.e. the mechanism should be anonymous: The probability that a player gets the object should only depend on the vector of values and not on the player's name, and similarly for the transfers. In particular, symmetry implies that the functions P_i, T_i and U_i do not depend on the player index i, hence, from now on, we will drop this index. Secondly, we want the allocation to be ex post efficient, i.e. the object should end up with a player who values it most, hence

$$\text{if } p_i(v) > 0, \text{ then } v_i = v^\wedge \tag{2.9}$$

A mechanism is *ex post efficient* if it satisfies (2.9) for all v. Finally, we want all players to be satisfied with the final allocation, i.e. the final allocation should be *envy free* (Foley (1967)): Ex post each player should (weakly) prefer what he himself receives above what is allocated to some other player. This requirement implies that any two players that do not receive the object get the same monetary transfer

$$t_{ij}(v) = t_{il}(v) = -t_{ii}(v)/(n-1) \qquad \text{for all } i, j, l \text{ with } j, l \neq i.$$

It also implies that if player i gets the object, he prefers to make the transfer, i.e.

$$v_i - t_{ii}(v) \geq t_{ii}(v)/(n-1) \qquad \text{for all } i, v \text{ with } p_i(v) > 0,$$

and that each player j not getting the object indeed prefers not to get it

$$t_{ii}(v)/(n-1) \geq v_j - t_{ii}(v) \qquad \text{for all } j \neq i, \text{ if } p_i(v) > 0.$$

Hence, we see that ex post efficiency is a necessary condition for ex post fairness. The final allocation is envy free if and only if the object is allocated efficiently and the winner pays each partner the same amount $\tau(v)$ with

$$v^s/n \leq \tau(v) \leq v^h/n \tag{2.10}$$

where v^h (resp. v^s) denotes the highest (resp. second highest) value. A mechanism that generates an envy free allocation for each possible value vector, will be called an *ex post fair mechanism*. The remainder of the paper is devoted to the question of whether such mechanisms exist and what their properties are.

3 Examples of Mechanisms

In this section several division methods that have been proposed in the literature will be illustrated. Attention will be confined to the case in which there are only two participants and in which F is the uniform distribution on $[0, 1]$.

A first procedure is random allocation: each player receives the object with probability 1/2 and there are no side payments. Clearly, this mechanism is very inefficient, in fact, the results of the next section imply that this mechanism yields the lowest expected utility for each player, no matter what his value might be. One method for improving the performance of this mechanism readily suggests itself, viz. let the random allocation be followed by bargaining between the partners. Since a player cannot be forced to trade if he doesn't want to, each player can only gain by engaging in the bargaining and, therefore, the expected payoffs will be higher. The final allocation (and, hence, the expected payoffs) will depend on how the rules for the bargaining game are specified. For example, suppose that the rules are that simultaneously the buyer and the seller submit bids and that the object is transferred, for a price equal to the average of the bids, if and only if the buyer's bid exceeds that of the seller. Chatterjee and Samuelson (1982) have shown that this game has an equilibrium[6] given by

$$\sigma_s(v_s) = 2/3\ v_s + 1/4 \qquad\qquad \sigma_b(v_b) = 2/3\ v_b + 1/12, \tag{3.1}$$

hence, in the range where trade is possible, the seller overstates his value while the buyer understates his, and this has the consequence that the outcome is not always ex post efficient. Straightforward computations show that the strategies from (3.1) yield the expected utility function U given by

$$U(v_i) = \begin{cases} 1/4\ v_i^2 + 1/8\ v_i + 9/64 & \text{if} \quad v_i \leq 1/4 \\ 1/2\ v_i^2 + 10/64 & \text{if} \quad 1/4 \leq v_i \leq 3/4 \\ 1/2\ v_i^2 + 3/8\ v_i + 1/64 & \text{if} \quad v_i \geq 3/4 \end{cases} \tag{3.2}$$

The reader might think that by using a different bargaining procedure, the performance of this type of mechanism can be improved. However, it follows from the results of Myerson and Satterthwaite (1983) that, no matter what the rules are, there will always be combinations of values for which the object ends up with the person who values it least. Random allocation followed by bargaining performs badly because players are treated asymmetrically: Once the initial property rights have been assigned, the partners have conflicting interests. In order to get a better price, the seller pretends that the object is worth more to him than it actually is while the buyer understates his value; strategic behavior which implies that the players may fail to strike an efficient bargain. These incentives for strategic manipulation can be reduced by forcing the players to announce their bids before the object is allocated; if a player doesn't know whether he will be the buyer or

[6]There exist other equilibria as well.

the seller, then his bid will be closer to the actual value.[7] Formally, one may proceed as follows. The players are asked to simultaneously submit bids b_1 and b_2, and then the object is randomly allocated. If the random move assigns the object to player i, then player i may retain the object if $b_i \geq b_j$, otherwise he sells it to player j for the price $p = (b_i + b_j)/2$. Because of risk neutrality, this mechanism is equivalent to the auction mechanism in which the players bid and the object is allocated to the highest bidder who pays his partner a compensation of $p = (b_1 + b_2)/4$. This auction mechanism will be analyzed at the end of this section (it corresponds to the auction with $\lambda = 1/2$) and we will see that the expected utilities generated by this mechanism dominate those given by (3.2).

Another mechanism that may be used is the divide and choose method, specifically the variant of this method that has been proposed in Luce and Raiffa (1957). Each person adds 1 (perfectly divisible) money unit to the pot, then a random move determines who will be the divider, this divider transfers a certain amount x from the pot to the object and his partner has the choice between the object plus x or the remainder, $2 - x$, of the money. If player d is the divider and he transfers $x \in [1/2, 1]$, then player c chooses the object if $v_c \geq 2 - 2x$, so that player d's expected payoff is

$$(2 - x)(2x - 1) + (v_d + x)(2 - 2x).$$

Hence, player d's optimal choice is

$$x_d = 7/8 - v_d/4$$

and the divider's equilibrium payoffs are

$$U_d(v_d) = (v_d/2 + 1/4)^2 \tag{3.3}$$

while those of the chooser are given by

$$U_c(v_c) = \begin{cases} 1/4 & \text{if } \; v_c \leq 1/4 \\ 1/4 + (v_c - 1/4)^2 & \text{if } \; 1/4 \leq v_c \leq 3/4 \\ v_c - 1/4 & \text{if } \; v_c \geq 3/4 \end{cases} \tag{3.4}$$

[7] This argument has also been made in Samuelson (1985). That paper also discussed the λ-auction mechanism with $\lambda = 1/2$.

It is interesting to note that $U_c(v_i) > U_d(v_i)$ for all v_i, hence, the method favors the chooser. This is in sharp contrast to the case in which the values are perfectly known. In the latter case, the method favors the divider, he can extract all the surplus. Also note that, under incomplete information, the object ends up with the chooser if and only if $v_c \geq 1/4 + v_d/2$, hence, also this method may lead to an inefficient allocation. Again the reason is strategic manipulation: If v_d is small (resp. large), then the divider is tempted to transfer relatively little (resp. relatively much) to the object, and if the chooser's value is below (resp. above) average, then the chooser takes the alternative that the divider doesn't "expect" and an inefficient outcome results.

The cause of the inefficiency associated with the divide and choose method is again the fact that players are in asymmetric roles. It is better not to introduce roles and keep the symmetry. There are various possibilities for modifying the method in this way. We now discuss two of these, both based on ideas of Banach and Knaster as reported in Steinhaus (1948). The essential idea is to let the division be performed by a mediator. For example, the mediator continuously transfers money from the pot to the object until one of the players shouts 'stop'. This player then receives the object plus the money that has been transferred, his partner receives the remainder of the money. It is clear that this mechanism is actually an auction mechanism. If we write $s_i = 1 + b_i/2$ for the value of the pot at which player i shouts 'stop', and interpret b_i as player i's bid, then the highest bidder gets the object and he pays his partner a price equal to half of his bid. (Hence, this is the special case of the auction mechanism introduced below with $\lambda = 1$.)

Alternatively, the mediator may first add all money to the object and then continuously transfer money from the object to the pot. In this case the person who first shouts 'stop' receives the pot, his partner receives the object plus the remainder of the money. Again this mechanism is actually an auction mechanism. If player i shouts 'stop' when the amount that has been transferred to the pot is s_i and we write $s_i = 1 + b_i/2$ then (with b_i being the bid of player i) the highest bidder gets the object and he pays this partner a price equal to half of the bid that the partner made. (Hence, this is the special case of the auction mechanism introduced next with $\lambda = 0$.)

Generally, an auction mechanism may be described as follows. Simultaneously the players bid and the object is allocated to the highest bidder for a price of λ times the highest bid, b_h, plus $1 - \lambda$ times the second highest bid, b_s. The revenues of the auction are shared equally by the partners so that, effectively, the winner pays his partner a compensation equal to $(\lambda b_h + (1 - \lambda)b_s)/2$. Above we encountered the Dutch auction ($\lambda = 1$), the Vickrey auction ($\lambda = 0$) and the case with $\lambda = 1/2$

as special cases. It is straightforward to verify that the strategy combination σ given by

$$\sigma_i(v_i) = 2/3v_i + 1/3(1-\lambda) \tag{3.5}$$

is a symmetric equilibrium of the auction mechanism. In Van Damme (1984) it was shown that this strategy pair is actually the unique symmetric equilibrium. Note that the equilibrium strategy is monotonic, hence, the person with the highest value bids highest so that the object always ends up with the person who values it most. Hence, any auction mechanism is ex post efficient[8]. The reader can easily verify that the expected payoffs associated with the auction mechanism are given by

$$U(v_i) = v_i^2/2 + 1/6. \tag{3.6}$$

Hence, the expected payoffs are independent of λ, a fact that is explained by the results of Sect. 4. The reader also notes that in terms of expected utilities the auction mechanisms indeed dominate random allocation followed by bargaining (the RHS from (3.6) is always strictly larger than that of (3.2)) as well as the divide and choose method (if U is as in (3.6), then $U(v_i) \geq U_d(v_i)/2 + U_c(v_i)/2$ with equality only for $v_i \in \{1/6, 5/6\}$). Although the different auction methods are equivalent at the interim stage (i.e. when a player only knows his own value), they are not equivalent ex post: the transfer payments depend on λ. If $v_1 > v_2$, then player 1 pays player 2 the amount $\lambda v_1/3 + (1-\lambda)(v_2/3 + 1/2)$. One notes that this transfer cannot always lie between $v_2/2$ and $v_1/2$ so that no auction mechanism is ex post fair.

The following two conclusions emerge from these examples. Mechanisms that have players play different roles typically lead to inefficient outcomes. Symmetric mechanisms have better efficiency properties, but also these mechanisms may yield outcomes that are not ex post fair. In the Sections 6 and 7 we will describe mechanisms that are ex post fair. In Sect. 4 we explain why all auction mechanisms are equivalent at the interim stage.

[8] In Van Damme (1984) it was shown that these properties hold for any number of players, n, and for any distribution F.

4 Revenue Equivalence

In this section we derive a 'revenue equivalence theorem' that explains why all auction mechanisms are equivalent at the interim stage. From now on attention will be restricted to direct mechanisms. By the revelation principle, this is without loss of generality.

We start by giving a characterization of incentive compatibility. Recall that a direct mechanism is said to be incentive compatible if truthtelling is a Nash equilibrium, i.e. if for all i and all $v_i, w_i \in [0,1]$

$$v_i P(v_i) - T(v_i) \geq v_i P(w_i) - T(w_i) \tag{4.1}$$

where P and T are defined by (2.5) and (2.6) and by the remarks concerning notation that were made after (2.8). The proof of Lemma 1 is standard and follows ideas outlined in Myerson and Satterthwaite (1983). (See also Cramton et al. (1987).) To make the paper self-contained, however, the proof is given in the appendix.

Lemma 1 . *A direct mechanism μ is incentive compatible if and only if P is nondecreasing and T is related to P according to*

$$T(v_i) = T(0) + \int_0^{v_i} x\,dP(x) \qquad (v_i \in [0,1]) \tag{4.2}$$

Since transfers sum to zero for each realized vector of values (Eq. (2.3)) and since players are treated symmetrically, we have that

$$\int T(v_i)\,dF(v_i) = 0. \tag{4.3}$$

Together with (4.2) this boundary condition implies that, for incentive compatible mechanisms, T and therefore U is completely determined by P. In particular it follows that all ex post efficient mechanisms are equivalent at the interim stage. (All such mechanisms have $P(v_i) = F(v_i)^{n-1}$.) Straightforward calculations show that

$$T(0) = \int_0^1 (1 - F(x) - xf(x))P(x)\,dx,$$

hence that

$$U(v_i) = \int_0^1 (F(x) + xf(x) - 1)P(x)dx + \int_0^{v_i} P(x)dx \tag{4.4}$$

For ex post efficient mechanisms, this expression can be rewritten as

$$U(v_i) = \frac{1}{n} + \frac{n-1}{n}\int_0^1 F^n(x)dx - \int_0^1 F^{n-1}(x)dx + \int_0^{v_i} F^{n-1}(x)dx \tag{4.5}$$

Let us remark that the constant term $U(0)$ in Eq. (4.5) is equal to $(1/n)$th of the expected revenue generated when the object is auctioned, say by the Vickrey procedure (or indeed by any auction procedure that always allocates the object to the person who values it most). We return to this property in Section 7. The following proposition summarizes the results obtained thus far

Proposition 1 . *The expected utility function U associated with an incentive compatible mechanism depends only on the probability function P and is given by (4.4). For ex post efficient mechanisms, $P(x) = F(x)^{n-1}$ and then the expected utility function is given by (4.5).*

Note that Lemma 1 and Proposition 1 imply that the expected utility function U is nondecreasing and convex. Another interesting property is that $U(v_i) \geq v_i/n$ for each mechanism and each value v_i, hence, each mechanism yields at least as much utility as random allocation does. Note that Proposition 2 implies that the individual rationality constraints are not binding, each player is, no matter what his value might be, willing to participate in any mechanism.

Proposition 2 . *Every mechanism is (weakly) preferred to random allocation, i.e. $U(v_i) \geq v_i/n$ for any incentive compatible mechanism.*

Proof. See the appendix.

From now on attention will be restricted to ex post efficient mechanisms. The following Proposition, which states that ex ante efficiency is equivalent to ex post efficiency, provides an additional justification for this restriction. Namely the proposition implies that there does not exist a mechanism that is uniformly preferred above an ex post efficient mechanism. A mechanism is said to be ex ante efficient if it maximizes the ex ante expected payoffs over the set of all incentive compatible mechanisms, i.e. if M is the set of all incentive compatible mechanisms satisfying (2.1) - (2.3), then $\nu \in M$ is *ex ante efficient* if

$$\int U_\mu(v_i)dF(v_i) \leq \int U_\nu(v_i)dF(v_i)$$

for all $\mu \in M$, where $U_\mu(v_i)$ is player i's expected payoff associated with μ if his value is v_i.

Proposition 3 . *The mechanism μ is ex ante efficient if and only if it yields an ex post efficient outcome for almost all value combinations, i.e. if (2.9) is satisfied for almost all v.*

Proof. Let ν be any ex post efficient mechanism[9]. Then for any mechanism μ we have in view of (4.3)

$$\int U_\mu(v_i)dF(v_i) = \int v_i P(v_i)dF(v_i)$$

Furthermore, with v^h being the highest component of v, we have

$$n\int v_i P(v_i)dF(v_i) = n\int v_i p_i(v)dF(v)$$

$$= \int \sum_i v_i p_i(v)dF(v) \leq \int v^h \sum_i p_i(v)dF(v)$$

$$= \int v^h dF(v) = n\int U_\nu(v_i)dF(v_i)$$

Since the inequality in the above chain is strict unless (2.9) is satisfied for almost all v, the proof is complete. □

5 An Impossibility Theorem

In this section we turn to some resolutions of the fair division problem that have been proposed in the literature for the case in which the value vector is commonly known, and we will show that the

[9]Such a mechanism indeed exists: In Van Damme (1984) it was shown that the λ-auction methods discussed in Section 3 have this property. Cramton et al. (1987) have shown that, more generally, ex post efficient mechanisms exist if and only if the initial distribution of ownership shares is not "too asymmetric".

allocations corresponding to these procedures cannot be implemented when there is incomplete information about the players' values.

A fair division method is a mapping m that assigns to each value vector v a utility $u_i^m(v)$ for each player i. For an incentive compatible direct mechanism μ let us write $u_i^\mu(v)$ for the utility that player i gets from μ when the value vector is v (assuming, of course, that the truthtelling equilibrium is played in μ). Hence $u_i^\mu(v) = v_i p_i(v) - t_i(v)$. We say that the fair division method m can be *implemented* if there exists an incentive compatible mechanism μ with $u_i^\mu(v) = u_i^m(v)$ for all i and v. We will show that neither the Steinhaus division method (Steinhaus (1948)), nor the egalitarian division can be implemented in case there is incomplete information. Since both methods can be obtained by applying the Nash bargaining solution (Nash (1950)) resulting from an appropriately chosen threat point, we turn to the Nash bargaining solution first.

Assume that the vector of values v is common knowledge. Since side payments are possible, the Pareto efficient frontier in utility space is given by

$$\{u \in \mathbf{R}^n;\ \sum u_i = v^h\}$$

If d is the utility vector in case of disagreement and $\sum_i d_i < v^h$, then the Nash bargaining solution of the problem is the utility vector u^* determined by

$$\sum u_i^* = v^h \quad \text{and} \quad u_i^* - d_i = u_1^* - d_1 \qquad\qquad \text{all } i$$

hence

$$u_i^* = d_i + (v^h - \sum d_j)/n \tag{5.1}$$

Two choices of d appear natural. If in case of conflict the object is destroyed then $d_i = 0$ for all i and $u_i^* = v^h/n$. This allocation corresponds to the egalitarian solution, the side payments are arranged in such a way that all players have equal payoffs. A second possibility is to resort to random allocation in case of conflict. Then $d_i = v_i/n$ for all i and the resulting allocation is the one proposed in Steinhaus (1948). Steinhaus calls v_i/n the 'fair share' of player i and he proposes to divide equally the surplus that remains after each player has received his fair share. Note that the Steinhaus allocation is not envy free.

The main result of this section is

Proposition 4 . *Neither the egalitarian allocation nor the Steinhaus allocation can be implemented when there is asymmetric information about the value vector v.*

Proof. For $\lambda \in [0,1]$ write

$$u_i^\lambda(v) = \lambda v_i/n + \left(v^\wedge - \lambda \sum v_j/n\right)/n \tag{5.2}$$

Then $\lambda = 0$ corresponds to the egalitarian solution while $\lambda = 1$ yields the Steinhaus allocation. We have to show that there does not exist an incentive compatible mechanism μ with $u_i^\mu(v) = v_i p_i(v) - t_i(v) = u_i^\lambda(v)$ for all v. Assume such a mechanism μ does exist. Then μ is ex post efficient so that, from Proposition 1, we get $U'(v_i) = P(v_i) = F(v_i)^{n-1}$. On the other hand, Eq. (5.2) yields

$$\begin{aligned} nU(v_i) &= n\int u_i^\lambda(v_{-i}, v_i)dF_{-i}(v_{-i}) \\ &= \frac{n-1}{n}\lambda v_i + v_i F(v_i)^{n-1} + \int_{v_i}^1 x dF(x)^{n-1} - (n-1)\lambda \int x dF(x)/n \end{aligned}$$

so that

$$nU'(v_i) = \frac{n-1}{n}\lambda + F(v_i)^{n-1}$$

Combining this expression with $U'(v_i) = F(v_i)^{n-1}$ yields

$$F^{n-1}(v_i) = \lambda/n,$$

hence, F is constant. But this contradicts the assumption that we have a situation with incomplete information. □

It is interesting to investigate which consequences strategic behavior has on the final outcome if the Steinhaus procedure or the egalitarian procedure is used to determine the allocation. Identifying a stated value with a bid, we see that the rules of the latter procedure correspond to those of an auction in which the object is allocated to the highest bidder who pays each of his partners $1/n$th of his bid. In general this method yields an ex post efficient outcome, but as we know from Section 3 the outcome need not be ex post fair. (The egalitarian method corresponds to the λ-auction from Section 3 with $\lambda = 1$.) The Steinhaus procedure corresponds to an auction in which the highest bidder (say this is player i) receives the object and in which this person pays each player j an amount equal to $b_j/n + (b_i - \sum_j b_j/n)/n$. Lyon (1986) has shown that this auction mechanism has a symmetric, increasing equilibrium. Hence, this mechanism is also ex post efficient and the interim expected payoffs are given by (4.5). In the case of 2 players these rules correspond to those of the auction mechanism from Section 3 with $\lambda = 1/2$, hence, the final outcome need not be fair.

6 A Possibility Theorem

In this section we show that it is possible to ensure that the final allocation will be envy free by giving an example of a mechanism leading to fair allocations.

Proposition 5 . *There exists an incentive compatible, ex post fair mechanism.*

Proof: For a value vector $v \in \mathbf{R}_n$ let v^h be the highest component of v and let v^s be the second highest component of v. Consider the following direct mechanism: The object is allocated to the person with the highest value, he pays the amount

$$t(v^h, v^s) = \frac{1}{n}\left\{v^h - \int_{v^s}^{v^h} F(x)dx\right\} \tag{6.1}$$

to each of his partners. In case there are multiple, say k, players with the highest value then each of them receives the object with probability $1/k$ and the person getting the object pays each of his partners v^h/n.

Clearly, this mechanism treats the players symmetrically. Furthermore, the amount that the winner pays to each of his partners lies inbetween v^s/n and v^h/n so that the mechanism is ex post fair. It remains to verify that the mechanism is incentive compatible, i.e. that truthtelling is a Nash equilibrium. Since the mechanism is ex post efficient, it suffices, by Lemma 1, to show that the expected transfer payments satisfy $T'(v_i) = v_i P'(v_i) = (n-1)v_i f(v_i)F(v_i)^{n-2}$. The verification of this identity involves straightforward calculations which are carried out in the appendix. □

Note that, since the mechanism from Proposition 3 is a direct mechanism, it is context dependent, i.e. the rules of the mechanism directly depend on the characteristics of the underlying uncertainty, i.e. the mechanism depends on the distribution function F. The author has not succeeded in finding a context independent mechanism of which the equilibrium gives rise to the direct mechanism from Proposition 3. It should also be noted that the mechanism described in (6.1) is probably not the unique ex post fair mechanism, however, in a certain sense it is the simplest one possible. Namely, it is the unique ex post fair mechanism for which the transfers depend only on v^h and v^s and for which

$$\frac{\partial t(v^h, v^s)}{\partial v^h} \text{ is independent of } v^s, \text{ and}$$
$$\frac{\partial t(v^h, v^s)}{\partial v^s} \text{ is independent of } v^h.$$

7 Dominant Strategy Mechanisms

Up to now we have focused on Bayesian Nash equilibria of mechanisms. It is easily seen that, without relaxing some of our requirements, no positive results can be obtained for the stronger notion of dominant strategy equilibria. Truthtelling is a dominant strategy for each player in the direct mechanism μ if

$$v_i p_i(v) - t_i(v) \geq v_i p_i(v \backslash w_i) - t_i(v \backslash w_i) \quad \text{for all } i, v, w_i \tag{7.1}$$

with the inequality being strict for at least one value vector v. For ex post efficient mechanisms, condition (7.1) implies that the winner's transfer payment should be independent of his value, and that also each looser's transfer must be independent of this player's value. In ex post fair mechanisms all loosers receive the same payoff, which by (7.1) can, therefore, only depend on the winner's value. Hence, if the transfers have to balance for each combination of values (condition (2.3)), then the transfers must be constant, but surely a mechanism in which the transfers do not depend on the value vector cannot be ex post fair. Hence,

Proposition 6 . *There does not exist an ex post fair mechanism for which truthtelling is an equilibrium in dominant strategies.*

If one insists on equilibria in dominant strategies, then positive results can be obtained by relaxing condition (2.3): One may be satisfied if the transfers balance on average, i.e. if condition (4.3) is satisfied instead of condition (2.3). Condition (4.3) may be established by allowing the mediator to act as a clearing house who balances the payments. (If mediators are risk neutral and if the market for mediators is competitive, the players will indeed be able to find a mediator who is willing to play this role.) By reviewing the proof of Proposition 1, one sees that condition (2.3) is not essential for this result to hold, the proof just uses (4.3). Hence, if the mediator has zero expected profits, then (in an incentive compatible mechanism) the player's expected utilities are still given by (4.4). To have an ex post fair mechanism with an equilibrium in dominant strategies, the winner should pay an amount $w(v)$ that does not depend on his own value, while each looser should get an amount $l(v)$ that only depends on the winner's value. Furthermore, it should be the case that $v^h - w(v) \geq l(v) \geq v^s - w(v)$. One possibility immediately suggests itself: The mediator first buys the object for a price B of which each player receives B/n; after the mediator has acquired it, he sells is again to one of the partners by using the Vickrey (1962) procedure, i.e. the object is allocated to the highest bidder who pays the second highest bid. Hence, using the above notation, $w(v) = -B/n + v^s$ and $l(v) = B/n$. Clearly, truthtelling is a dominant strategy and the resulting allocation is always fair. Furthermore, in light of the remarks made above, the mediator's expected payoffs are zero if his bid B is equal to $nU(0)$ where $U(0)$ is given by (4.5).

An alternative possibility is that the mediator uses the information revealed by the auction to determine the compensation of the loosers rather than to determine the price that the winner should pay. Specifically, the mediator may use the following procedure. First he asks each player to contribute an amount C equal to $(n-1)/n$ times the expectation of the highest value (i.e. $C = (n-1)\int v^h dF(v)/n$). After these contributions have been made he auctions the object. This time, however, the winner does not have to pay anything, rather it is the case that each looser receives an amount equal to the winner's value. Hence, in terms of the above notation, $w(v) = C$ and $l(v) = v^h - C$. Again truthtelling is a dominant strategy and, no matter what the values are, all players have the same net payoffs. Hence we have shown

Proposition 7 . *With an active mediator, who is used to balance the budgets on average, there exists an ex post fair mechanism for which truthtelling is an equilibrium in dominant strategies. In fact there exists such a mechanism that leads to the egalitarian outcome, i.e. all players have the same net payoff.*

Clearly, the mechanisms discussed in this section are viable only if the mediator is able to make an accurate assessment of the value of the object, i.e. if he knows the distribution F. It will also be clear that even if he knows F, he will be reluctant to use the second procedure discussed above. Namely, although truthtelling is a dominant strategy in the game, the mediator should fear that the agents will try to increase their payoffs by more sophisticated ways of manipulating the outcome. For example, the players could make a secret contract that each player will bid his value plus an amount x and that each looser will pay the winner x/n. With this contract in place it is a dominant strategy for each player i to bid $v_i + x$ in the mediator's auction game. Compared with the original situation in which there is no contract each player increases his payoff by $(n-1)x/n$, at the expense of the mediator who makes an expected loss of $(n-1)x$. Hence, it is very unlikely that we will observe such a mechanism in practice.

8 Conclusion

In this paper we have investigated a simple fair division situation with incomplete information. The analysis was simplified by the assumptions of symmetry (all values are drawn from the same distribution) and independence (all values are drawn independently). Further research should be devoted to relaxing these assumption. The assumption of independence seems especially inappropriate in the case of the dissolving of joint ventures, one of the examples that was mentioned in the Introduction. In that case the value of each partner depends on the future prospects for the business about which the partners may have different information.

The paper was motivated by the idea that it is desirable to guarantee an ex post fair outcome. It may be questioned whether ex post fairness is indeed desirable, especially since some types of a player may, at the interim stage, prefer non-fair mechanisms above fair ones[10]. In Van Damme (1985) it was shown that, for the example discussed in Section 3, the expected utility for a player with value 1/2 is maximized by a mechanism that assigns the object to the person with the highest value when this value is not in the interval $[1/4, 3/4]$ and that allocates the object randomly if the highest value is in this interval. In that paper it was also shown that only a type with value $v_i = 0$ or $v_i = 1$ prefers an ex post fair mechanism above any other mechanism. If one does

[10] Why this is so can easily be seen in an asymmetric example. Suppose that player 1 values the object at $v_1 = 0$ and that player 2 values the object either at $v_2 = 0$ (with probability p) or at $v_2 = 100$ (with probability $1-p$). In an ex post fair incentive compatible mechanism, player 2 always gets the object and he pays at most 5 for it. If p is sufficiently small player 1 clearly prefers a mechanism in which the object is allocated only to player 2 if this player is willing to pay a sufficiently high price for it.

not insist on ex post efficiency, then the allocation mechanism should reflect a fair compromise between the alternative types of a player[11] and the solutions of Harsanyi and Selten (1972) and Myerson (1984) specify axioms for determining such fair compromises. If, however, one insists on ex post efficiency, then Proposition 1 shows that the interim utilities are completely determined so that according to Harsanyi/Selten and Myerson all mechanisms are equivalent. At the ex post stage, however, such mechanisms are not necessarily equivalent: some of these guarantee ex post fair mechanisms while others do not.

References

Chatterjee, K. and W. Samuelson (1983). "Bargaining under Incomplete Information", *Operations Research*, **31**, 838-851.

Cramton, P., R. Gibbons and P. Klemperer (1987). "Dissolving a Partnership Efficiently", *Econometrica*, **55**, 615-632.

Crawford, V. (1977). "A Game of Fair Division", *Review of Economic Studies*, **44**, 235-247.

Crawford, V. and W. Heller (1979). "Fair Division with Indivisible Commodities", *Journal of Economic Theory*, **21**, 10-27.

Dubins, L. (1977). "Group Decision Devices", *American Mathematical Monthly*, **84**, 350-356.

Foley, D. (1967). "Resource Allocation and the Public Sector", *Yale Economic Essays*, **7**.

Güth, W. (1986). " Auctions, Public Tenders, and Fair Division Games: An Axiomatic Approach", *Mathematical Social Sciences*, **11**, 283-294.

Güth, W. and E. van Damme (1986). "A Comparison of Pricing Rules for Auctions and Fair Division Games", *Social Choice and Welfare*, **3**, 177-198.

Harsanyi, J. (1967-8). "Games with Incomplete Information Played by 'Bayesian' players", *Management Science*, **14**, 159-189, 320-344, 486-504.

Harsanyi, J. and R. Selten (1972). "A Generalized Nash Solution for Two-Person Bargaining Games with Incomplete Information", *Management Science*, **18**, 80-105.

Kennan, J. and R. Wilson (1990). "Bargaining with Private Information", *SITE Technical report*, **5**, Stanford University.

Kuhn, H. (1967). "On Games of Fair Division", in *Essays in Mathematical Economics*

[11] In our symmetric setup there is no conflict between different players.

in Honor of Oskar Morgenstern, ed. M. Shubik, Princeton University Press, Princeton.

Luce, R. and H. Raiffa (1957). *Games and Decisions*, Wiley, New York.

Lyon, R. (1986). "Equilibrium Properties of Auctions and Alternative Procedures for Allocating Transferable Permits", *Journal of Environmental Economics and Management*, **13**, 129-152.

Myerson, R. (1979). "Incentive Compatibility and the Bargaining Problem", *Econometrica*, **47**, 61-73.

Myerson, R. (1984a). "Two-Person Bargaining Problems with Incomplete Information", *Econometrica*, **52**, 461-487.

Myerson, R. (1984b). "Cooperative Games with Incomplete Information", *International Journal of Game Theory*, **14**, 69-97.

Myerson R. and M. Satterthwaite (1983). " Efficient Mechanisms for Bilateral Trading", *Journal of Economic Theory*, **29**, 265-281.

Nash, J. (1950). "The Bargaining Problem", *Econometrica*, **18**, 155-162.

Pazner, E. and D. Schmeidler (1978). "Egalitarian Equivalent Allocations: A New Concept of Economic Equity", *Quarterly Journal of Economics*, **92**, 671-687.

Samuelson, W. (1980). "The Object Distribution Problem Revisited", *Quarterly Journal of Economics*, **94**, 85-98.

Samuelson, W. (1985). "A comment on the Coase Theorem". In A.E. Roth (Ed.) *Game-theoretic Models of Bargaining*. Cambridge University Press, Cambridge, 321-340.

Steinhaus, H. (1948). "The Problem of Fair Division", *Econometrica*, **16**, 101-104.

Van Damme, E. (1984). "Auctions and Fair Division", Mimeo, Dept. of Mathematics and Computer Science, University of Technology, Delft.

Van Damme, E. (1985). "Fair Allocation of an Indivisible Commodity", Mimeo, Dept. of Mathematics and Computer Science, University of Technology, Delft.

Varian, H. (1974). "Equity, Envy and Efficiency", *Journal of Economic Theory*, **9**, 63-90.

Vickrey, W. (1961). "Counterspeculation, Auctions and Competitive Sealed Tenders", *Journal of Finance*, **16**, 8-37.

Appendix

Proof of Lemma 1. If μ is incentive compatible, then (4.1) holds and by rearranging and interchanging the roles of v_i and w_i we obtain

$$w_i(P(v_i) - P(w_i)) \leq T(v_i) - T(w_i) \leq v_i(P(v_i) - P(w_i)),$$

so that P must be nondecreasing. Furthermore, T is differentiable whenever P is, and at such points of differentiability $T'(v_i) = v_i P'(v_i)$. At other points, a jump in $v_i P(v_i)$ is matched by a jump in $T(v_i)$ and such discontinuities are accounted for by in the integral in (4.2), which is understood to be a Lebesgue-Stieltjes integral. Hence, (4.2) is indeed correct.

Conversely, if (4.2) holds, then for any $v_i, w_i \in [0,1]$

$$T(v_i) - T(w_i) = \int_{w_i}^{v_i} x dP(x) \leq \int_{w_i}^{v_i} v_i dP(x) = v_i(P(v_i) - P(w_i)),$$

which, after rearranging, yields the incentive compatibility condition (4.1). □

Proof of Proposition 2. Because of symmetry, the ex ante probability that a player receives the object is $1/n$, hence $\int P(x)dF(x) = 1/n$. In view of (4.4) we therefore have to show that

$$v_i \int_0^1 f(x)P(x)dx \leq \int_0^1 (F(x) + xf(x) - 1)P(x)dx + \int_0^{v_i} P(x)dx,$$

or, equivalently

$$\int_0^{v_i} v_i f(x)P(x)dx + \int_{v_i}^1 (v_i f(x) + 1)P(x)dx \leq \int_0^1 (F(x) + xf(x))P(x)dx$$

Now by using partial integration it is easily seen that

$$\int_0^{v_i} (v_i - x)f(x)P(x)dx \leq \int_0^{v_i} F(x)P(x)dx,$$

so that it suffices to show that

$$\int_{v_i}^1 (v_i f(x) + 1)P(x)dx \leq \int_{v_i}^1 (F(x) + xf(x))P(x)dx,$$

or equivalently

$$\int_{v_i}^{1}(1-F(x))P(x)dx \leq \int_{v_i}^{1}(x-v_i)f(x)P(x)dx.$$

Integrating the integral of the RHS by parts we see that we have to show that

$$\int_{v_i}^{1}(1-F(x))P(x)dx \leq (1-v_i)P(1)-\int_{v_i}^{1}F(x)P(x)dx-\int_{v_i}^{1}(x-v_i)F(x)dP(x)$$

which is equivalent to

$$\int_{v_i}^{1}P(x)dx \leq (1-v_i)P(1)-\int_{v_i}^{1}(x-v_i)F(x)dP(x)$$

By integrating the integral of the LHS by parts it is seen that this inequality is equivalent to

$$v_i(P(1)-P(v_i))-\int_{v_i}^{1}x\,dP(x) \leq -\int_{v_i}^{1}(x-v_i)F(x)dp(x),$$

which in turn is equivalent to

$$\int_{v_i}^{1}(x-v_i)(1-F(x))dP(x) \geq 0,$$

an inequality which is clearly satisfied. □

Proof of Proposition 5: It remains to be shown that the mechanism determined by (6.1) is incentive compatible, i.e. that $T'(v_i)=v_iP'(v_i)=(n-1)v_if(v_i)F(v_i)^{n-2}$. Write $x=v_{-i}^{h}$ (resp. $y=v_{-i}^{s}$) for the highest (resp. second highest) component of v_{-i}. Then the joint distribution Φ of x and y is given by

$$\Phi(x,y)=\begin{cases} F^{n-1}(x)+(n-1)F^{n-2}(y)(F(x)-F(y)) & \text{if } x>y \\ F^{n-1}(y) & \text{if } x\leq y, \end{cases}$$

and the associated joint density is equal to

$$\varphi(x,y)=\begin{cases} (n-1)(n-2)f(x)f(y)F(y)^{n-3} & \text{if } x>y \\ 0 & \text{if } x\leq y \end{cases}$$

With t given by (5.1), the transfer that player i makes if he has the value v_i is equal to

$$t_i(v_i) = \begin{cases} (n-1)\, t(v_i, x) & \text{if } x < v_i \\ -t(x, v_i) & \text{if } y < v_i < x \\ -t(x, y) & \text{if } v_i > y, \end{cases}$$

and the expected transfer is

$$T(v_i) = \int\int t_i(v_i) d\Phi(x, y).$$

Differentiating with respect to v_i we find that

$$\begin{aligned} T'(v_i) &= (n-1)\int_0^{v_i}\int_0^{v_i} \frac{\partial}{\partial v_i} t(v_i, x) d\Phi(x, y) + (n+1)^2\, t(v_i, v_i) f(v_i) F(v_i)^{n-2} \\ &- \int_{v_i}^1 \int_0^{v_i} \frac{\partial}{\partial v_i} t(x, v_i) d\Phi(x, y) - \int_{v_i}^1 t(x, v_i) d\Phi(x, v_i) \\ &+ (n-1)\, t(v_i, v_i) f(v_i) F(v_i)^{n-2} + \int_{v_i}^1 t(x, v_i) d\Phi(x, v_i). \end{aligned}$$

Rearranging and collecting terms we see that

$$\begin{aligned} T'(v_i) &= (n-1) v_i\, f(v_i) F(v_i)^{n-2} + \frac{(n-1)}{n}(1 - F(v_i)) F(v_i)^{n-1} \\ &= \frac{n-1}{n} \int_{v_i}^1 \int_0^{v_i} F(v_i) d\Phi(x, y) \\ &= (n-1) v_i\, f(v_i) F(v_i)^{n-2}, \end{aligned}$$

and this is what had to be proved. □

Reprint Series, CentER, Tilburg University, The Netherlands:

No. 1 G. Marini and F. van der Ploeg, Monetary and fiscal policy in an optimising model with capital accumulation and finite lives, *The Economic Journal*, vol. 98, no. 392, 1988, pp. 772 - 786.

No. 2 F. van der Ploeg, International policy coordination in interdependent monetary economies, *Journal of International Economics*, vol. 25, 1988, pp. 1 - 23.

No. 3 A.P. Barten, The history of Dutch macroeconomic modelling (1936-1986), in W. Driehuis, M.M.G. Fase and H. den Hartog (eds.), *Challenges for Macroeconomic Modelling*, Contributions to Economic Analysis 178, Amsterdam: North-Holland, 1988, pp. 39 - 88.

No. 4 F. van der Ploeg, Disposable income, unemployment, inflation and state spending in a dynamic political-economic model, *Public Choice*, vol. 60, 1989, pp. 211 - 239.

No. 5 Th. ten Raa and F. van der Ploeg, A statistical approach to the problem of negatives in input-output analysis, *Economic Modelling*, vol. 6, no. 1, 1989, pp. 2 - 19.

No. 6 E. van Damme, Renegotiation-proof equilibria in repeated prisoners' dilemma, *Journal of Economic Theory*, vol. 47, no. 1, 1989, pp. 206 - 217.

No. 7 C. Mulder and F. van der Ploeg, Trade unions, investment and employment in a small open economy: a Dutch perspective, in J. Muysken and C. de Neubourg (eds.), *Unemployment in Europe*, London: The Macmillan Press Ltd, 1989, pp. 200 - 229.

No. 8 Th. van de Klundert and F. van der Ploeg, Wage rigidity and capital mobility in an optimizing model of a small open economy, *De Economist*, vol. 137, nr. 1, 1989, pp. 47 - 75.

No. 9 G. Dhaene and A.P. Barten, When it all began: the 1936 Tinbergen model revisited, *Economic Modelling*, vol. 6, no. 2, 1989, pp. 203 - 219.

No. 10 F. van der Ploeg and A.J. de Zeeuw, Conflict over arms accumulation in market and command economies, in F. van der Ploeg and A.J. de Zeeuw (eds.), *Dynamic Policy Games in Economics*, Contributions to Economic Analysis 181, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 91 - 119.

No. 11 J. Driffill, Macroeconomic policy games with incomplete information: some extensions, in F. van der Ploeg and A.J. de Zeeuw (eds.), *Dynamic Policy Games in Economics*, Contributions to Economic Analysis 181, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 289 - 322.

No. 12 F. van der Ploeg, Towards monetary integration in Europe, in P. De Grauwe et al., *De Europese Monetaire Integratie: vier visies*, Wetenschappelijke Raad voor het Regeringsbeleid V 66, 's-Gravenhage: SDU uitgeverij, 1989, pp. 81 - 106.

No. 13 R.J.M. Alessie and A. Kapteyn, Consumption, savings and demography, in A. Wenig, K.F. Zimmermann (eds.), *Demographic Change and Economic Development*, Berlin/Heidelberg: Springer-Verlag, 1989, pp. 272 - 305.

No. 14 A. Hoque, J.R. Magnus and B. Pesaran, The exact multi-period mean-square forecast error for the first-order autoregressive model, *Journal of Econometrics*, vol. 39, no. 3, 1988, pp. 327 - 346.

No. 15 R. Alessie, A. Kapteyn and B. Melenberg, The effects of liquidity constraints on consumption: estimation from household panel data, *European Economic Review*, vol. 33, no. 2/3, 1989, pp. 547 - 555.

No. 16 A. Holly and J.R. Magnus, A note on instrumental variables and maximum likelihood estimation procedures, *Annales d'Économie et de Statistique*, no. 10, April-June, 1988, pp. 121 - 138.

No. 17 P. ten Hacken, A. Kapteyn and I. Woittiez, Unemployment benefits and the labor market, a micro/macro approach, in B.A. Gustafsson and N. Anders Klevmarken (eds.), *The Political Economy of Social Security*, Contributions to Economic Analysis 179, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 143 - 164.

No. 18 T. Wansbeek and A. Kapteyn, Estimation of the error-components model with incomplete panels, *Journal of Econometrics*, vol. 41, no. 3, 1989, pp. 341 - 361.

No. 19 A. Kapteyn, P. Kooreman and R. Willemse, Some methodological issues in the implementation of subjective poverty definitions, *The Journal of Human Resources*, vol. 23, no. 2, 1988, pp. 222 - 242.

No. 20 Th. van de Klundert and F. van der Ploeg, Fiscal policy and finite lives in interdependent economies with real and nominal wage rigidity, *Oxford Economic Papers*, vol. 41, no. 3, 1989, pp. 459 - 489.

No. 21 J.R. Magnus and B. Pesaran, The exact multi-period mean-square forecast error for the first-order autoregressive model with an intercept, *Journal of Econometrics*, vol. 42, no. 2, 1989, pp. 157 - 179.

No. 22 F. van der Ploeg, Two essays on political economy: (i) The political economy of overvaluation, *The Economic Journal*, vol. 99, no. 397, 1989, pp. 850 - 855; (ii) Election outcomes and the stockmarket, *European Journal of Political Economy*, vol. 5, no. 1, 1989, pp. 21 - 30.

No. 23 J.R. Magnus and A.D. Woodland, On the maximum likelihood estimation of multivariate regression models containing serially correlated error components, *International Economic Review*, vol. 29, no. 4, 1988, pp. 707 - 725.

No. 24 A.J.J. Talman and Y. Yamamoto, A simplicial algorithm for stationary point problems on polytopes, *Mathematics of Operations Research*, vol. 14, no. 3, 1989, pp. 383 - 399.

No. 25 E. van Damme, Stable equilibria and forward induction, *Journal of Economic Theory*, vol. 48, no. 2, 1989, pp. 476 - 496.

No. 26 A.P. Barten and L.J. Bettendorf, Price formation of fish: An application of an inverse demand system, *European Economic Review*, vol. 33, no. 8, 1989, pp. 1509 - 1525.

No. 27 G. Noldeke and E. van Damme, Signalling in a dynamic labour market, *Review of Economic Studies*, vol. 57 (1), no. 189, 1990, pp. 1 - 23.

No. 28 P. Kop Jansen and Th. ten Raa, The choice of model in the construction of input-output coefficients matrices, *International Economic Review*, vol. 31, no. 1, 1990, pp. 213 - 227.

No. 29 F. van der Ploeg and A.J. de Zeeuw, Perfect equilibrium in a model of competitive arms accumulation, *International Economic Review*, vol. 31, no. 1, 1990, pp. 131 - 146.

No. 30 J.R. Magnus and A.D. Woodland, Separability and aggregation, *Economica*, vol. 57, no. 226, 1990, pp. 239 - 247.

No. 31 F. van der Ploeg, International interdependence and policy coordination in economies with real and nominal wage rigidity, *Greek Economic Review*, vol. 10, no. 1, June 1988, pp. 1 - 48.

No. 32 E. van Damme, Signaling and forward induction in a market entry context, *Operations Research Proceedings 1989*, Berlin-Heidelberg: Springer-Verlag, 1990, pp. 45 - 59.

No. 33 A.P. Barten, Toward a levels version of the Rotterdam and related demand systems, *Contributions to Operations Research and Economics*, Cambridge: MIT Press, 1989, pp. 441 - 465.

No. 34 F. van der Ploeg, International coordination of monetary policies under alternative exchange-rate regimes, in F. van der Ploeg (ed.), *Advanced Lectures in Quantitative Economics*, London-Orlando: Academic Press Ltd., 1990, pp. 91 - 121.

No. 35 Th. van de Klundert, On socioeconomic causes of 'wait unemployment', *European Economic Review*, vol. 34, no. 5, 1990, pp. 1011 - 1022.

No. 36 R.J.M. Alessie, A. Kapteyn, J.B. van Lochem and T.J. Wansbeek, Individual effects in utility consistent models of demand, in J. Hartog, G. Ridder and J. Theeuwes (eds.), *Panel Data and Labor Market Studies*, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1990, pp. 253 - 278.

No. 37 F. van der Ploeg, Capital accumulation, inflation and long-run conflict in international objectives, *Oxford Economic Papers*, vol. 42, no. 3, 1990, pp. 501 - 525.

No. 38 Th. Nijman and F. Palm, Parameter identification in ARMA Processes in the presence of regular but incomplete sampling, *Journal of Time Series Analysis*, vol. 11, no. 3, 1990, pp. 239 - 248.

No. 39 Th. van de Klundert, Wage differentials and employment in a two-sector model with a dual labour market, *Metroeconomica*, vol. 40, no. 3, 1989, pp. 235 - 256.

No. 40 Th. Nijman and M.F.J. Steel, Exclusion restrictions in instrumental variables equations, *Econometric Reviews*, vol. 9, no. 1, 1990, pp. 37 - 55.

No. 41 A. van Soest, I. Woittiez and A. Kapteyn, Labor supply, income taxes, and hours restrictions in the Netherlands, *Journal of Human Resources*, vol. 25, no. 3, 1990, pp. 517 - 558.

No. 42 Th.C.M.J. van de Klundert and A.B.T.M. van Schaik, Unemployment persistence and loss of productive capacity: a Keynesian approach, *Journal of Macroeconomics*, vol. 12, no. 3, 1990, pp. 363 - 380.

No. 43 Th. Nijman and M. Verbeek, Estimation of time-dependent parameters in linear models using cross-sections, panels, or both, *Journal of Econometrics*, vol. 46, no. 3, 1990, pp. 333 - 346.

No. 44 E. van Damme, R. Selten and E. Winter, Alternating bid bargaining with a smallest money unit, *Games and Economic Behavior*, vol. 2, no. 2, 1990, pp. 188 - 201.

No. 45 C. Dang, The D_1-triangulation of $\mathbf{R}^n$ for simplicial algorithms for computing solutions of nonlinear equations, *Mathematics of Operations Research*, vol. 16, no. 1, 1991, pp. 148 - 161.

No. 46 Th. Nijman and F. Palm, Predictive accuracy gain from disaggregate sampling in ARIMA models, *Journal of Business & Economic Statistics*, vol. 8, no. 4, 1990, pp. 405 - 415.

No. 47 J.R. Magnus, On certain moments relating to ratios of quadratic forms in normal variables: further results, *Sankhya: The Indian Journal of Statistics*, vol. 52, series B, part. 1, 1990, pp. 1 - 13.

No. 48 M.F.J. Steel, A Bayesian analysis of simultaneous equation models by combining recursive analytical and numerical approaches, *Journal of Econometrics*, vol. 48, no. 1/2, 1991, pp. 83 - 117.

No. 49 F. van der Ploeg and C. Withagen, Pollution control and the ramsey problem, *Environmental and Resource Economics*, vol. 1, no. 2, 1991, pp. 215 - 236.

No. 50 F. van der Ploeg, Money and capital in interdependent economies with overlapping generations, *Economica*, vol. 58, no. 230, 1991, pp. 233 - 256.

No. 51 A. Kapteyn and A. de Zeeuw, Changing incentives for economic research in the Netherlands, *European Economic Review*, vol. 35, no. 2/3, 1991, pp. 603 - 611.

No. 52 C.G. de Vries, On the relation between GARCH and stable processes, *Journal of Econometrics*, vol. 48, no. 3, 1991, pp. 313 - 324.

No. 53 R. Alessie and A. Kapteyn, Habit formation, interdependent preferences and demographic effects in the almost ideal demand system, *The Economic Journal*, vol. 101, no. 406, 1991, pp. 404 - 419.

No. 54 W. van Groenendaal and A. de Zeeuw, Control, coordination and conflict on international commodity markets, *Economic Modelling*, vol. 8, no. 1, 1991, pp. 90 - 101.

No. 55 F. van der Ploeg and A.J. Markink, Dynamic policy in linear models with rational expectations of future events: A computer package, *Computer Science in Economics and Management*, vol. 4, no. 3, 1991, pp. 175 - 199.

No. 56 H.A. Keuzenkamp and F. van der Ploeg, Savings, investment, government finance, and the current account: The Dutch experience, in G. Alogoskoufis, L. Papademos and R. Portes (eds.), *External Constraints on Macroeconomic Policy: The European Experience*, Cambridge: Cambridge University Press, 1991, pp. 219 - 263.

No. 57 Th. Nijman, M. Verbeek and A. van Soest, The efficiency of rotating-panel designs in an analysis-of-variance model, *Journal of Econometrics*, vol. 49, no. 3, 1991, pp. 373 - 399.

No. 58 M.F.J. Steel and J.-F. Richard, Bayesian multivariate exogeneity analysis - an application to a UK money demand equation, *Journal of Econometrics*, vol. 49, no. 1/2, 1991, pp. 239 - 274.

No. 59 Th. Nijman and F. Palm, Generalized least squares estimation of linear models containing rational future expectations, *International Economic Review*, vol. 32, no. 2, 1991, pp. 383 - 389.

No. 60 E. van Damme, Equilibrium selection in 2 x 2 games, *Revista Espanola de Economia*, vol. 8, no. 1, 1991, pp. 37 - 52.

No. 61 E. Bennett and E. van Damme, Demand commitment bargaining: the case of apex games, in R. Selten (ed.), *Game Equilibrium Models III - Strategic Bargaining*, Berlin: Springer-Verlag, 1991, pp. 118 - 140.

No. 62 W. Güth and E. van Damme, Gorby games - a game theoretic analysis of disarmament campaigns and the defense efficiency - hypothesis -, in R. Avenhaus, H. Karkar and M. Rudnianski (eds.), *Defense Decision Making - Analytical Support and Crisis Management*, Berlin: Springer-Verlag, 1991, pp. 215 - 240.

No. 63 A. Roell, Dual-capacity trading and the quality of the market, *Journal of Financial Intermediation*, vol. 1, no. 2, 1990, pp. 105 - 124.

No. 64 Y. Dai, G. van der Laan, A.J.J. Talman and Y. Yamamoto, A simplicial algorithm for the nonlinear stationary point problem on an unbounded polyhedron, *Siam Journal of Optimization*, vol. 1, no. 2, 1991, pp. 151 - 165.

No. 65 M. McAleer and C.R. McKenzie, Keynesian and new classical models of unemployment revisited, *The Economic Journal*, vol. 101, no. 406, 1991, pp. 359 - 381.

No. 66 A.J.J. Talman, General equilibrium programming, *Nieuw Archief voor Wiskunde*, vol. 8, no. 3, 1990, pp. 387 - 397.

No. 67 J.R. Magnus and B. Pesaran, The bias of forecasts from a first-order autoregression, *Econometric Theory*, vol. 7, no. 2, 1991, pp. 222 - 235.

No. 68 F. van der Ploeg, Macroeconomic policy coordination issues during the various phases of economic and monetary integration in Europe, *European Economy - The Economics of EMU*, Commission of the European Communities, special edition no. 1, 1991, pp. 136 - 164.

No. 69 H. Keuzenkamp, A precursor to Muth: Tinbergen's 1932 model of rational expectations, *The Economic Journal*, vol. 101, no. 408, 1991, pp. 1245 - 1253.

No. 70 L. Zou, The target-incentive system vs. the price-incentive system under adverse selection and the ratchet effect, *Journal of Public Economics*, vol. 46, no. 1, 1991, pp. 51 - 89.

No. 71 E. Bomhoff, Between price reform and privatization: Eastern Europe in transition, *Finanzmarkt und Portfolio Management*, vol. 5, no. 3, 1991, pp. 241 - 251.

No. 72 E. Bomhoff, Stability of velocity in the major industrial countries: a Kalman filter approach, *International Monetary Fund Staff Papers*, vol. 38, no. 3, 1991, pp. 626 - 642.

No. 73 E. Bomhoff, Currency convertibility: when and how? A contribution to the Bulgarian debate, *Kredit und Kapital*, vol. 24, no. 3, 1991, pp. 412 - 431.

No. 74 H. Keuzenkamp and F. van der Ploeg, Perceived constraints for Dutch unemployment policy, in C. de Neubourg (ed.), *The Art of Full Employment - Unemployment Policy in Open Economies*, Contributions to Economic Analysis 203, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1991, pp. 7 - 37.

No. 75 H. Peters and E. van Damme, Characterizing the Nash and Raiffa bargaining solutions by disagreement point axions, *Mathematics of Operations Research*, vol. 16, no. 3, 1991, pp. 447 - 461.

No. 76 P.J. Deschamps, On the estimated variances of regression coefficients in misspecified error components models, *Econometric Theory*, vol. 7, no. 3, 1991, pp. 369 - 384.

No. 77 A. de Zeeuw, Note on 'Nash and Stackelberg solutions in a differential game model of capitalism', *Journal of Economic Dynamics and Control*, vol. 16, no. 1, 1992, pp. 139 - 145.

No. 78 J.R. Magnus, On the fundamental bordered matrix of linear estimation, in F. van der Ploeg (ed.), *Advanced Lectures in Quantitative Economics*, London-Orlando: Academic Press Ltd., 1990, pp. 583 - 604.

No. 79 F. van der Ploeg and A. de Zeeuw, A differential game of international pollution control, *Systems and Control Letters*, vol. 17, no. 6, 1991, pp. 409 - 414.

No. 80 Th. Nijman and M. Verbeek, The optimal choice of controls and pre-experimental observations, *Journal of Econometrics*, vol. 51, no. 1/2, 1992, pp. 183 - 189.

No. 81 M. Verbeek and Th. Nijman, Can cohort data be treated as genuine panel data?, *Empirical Economics*, vol. 17, no. 1, 1992, pp. 9 - 23.

No. 82 E. van Damme and W. Güth, Equilibrium selection in the Spence signaling game, in R. Selten (ed.), *Game Equilibrium Models II - Methods, Morals, and Markets*, Berlin: Springer-Verlag, 1991, pp. 263 - 288.

No. 83 R.P. Gilles and P.H.M. Ruys, Characterization of economic agents in arbitrary communication structures, *Nieuw Archief voor Wiskunde*, vol. 8, no. 3, 1990, pp. 325 - 345.

No. 84 A. de Zeeuw and F. van der Ploeg, Difference games and policy evaluation: a conceptual framework, *Oxford Economic Papers*, vol. 43, no. 4, 1991, pp. 612 - 636.

No. 85 E. van Damme, Fair division under asymmetric information, in R. Selten (ed.), *Rational Interaction - Essays in Honor of John C. Harsanyi*, Berlin/Heidelberg: Springer-Verlag, 1992, pp. 121 - 144.

P.O. BOX 90153 5000 LE TILBURG THE NETHERLANDS